# Obtaining solar eclipse and planetary transit contact times using DSLR imaging

Giovanni Di Giovanni

Based on a mathematical analysis of a sequence of DSLR images taken during the events, a simple method is presented to determine the times of contact of solar eclipses and planetary transits across the solar disk.

## Introduction

Accurate timings of eclipses and planetary transits were very common in the scientific literature of the past.[1–5] Such painstaking work was once invaluable in refining our knowledge of the lunar and terrestrial orbits, but today there is much less interest in these kind of measurements as new techniques have emerged. Nonetheless, eclipses continue to be timed with great accuracy at the Colle Leone Astronomical Observatory in Italy (MPC observatory code C96), in order to obtain exact times of contacts and event maxima. Such timing determinations were, for example, made during the 2015 March 20 solar eclipse, which was partial from our observing station (Figures 1a, 1b).

Increasingly, DSLR cameras are used to record eclipses. We have therefore developed a simple method, mainly for educational purposes, to determine the eclipse times of contact from an accurately timed sequence of DSLR images, which we shall describe in this paper.

## Recording and analysing the DSLR images

We obtained a sequence of DSLR images throughout the 2015 March 20 solar eclipse. A correction to UTC was made to the time recorded by the camera for each consecutive image by means of a radio-controlled clock. We estimate that the uncertainty on the timing of each image is ±0.5s. Using Photoshop software, we measured on each resulting image the length, in pixels, of the chord *C* common to both the solar and the lunar disks, and the distance, *V*, between the vertices of the visible arcs of the Moon and the Sun (Figure 2).

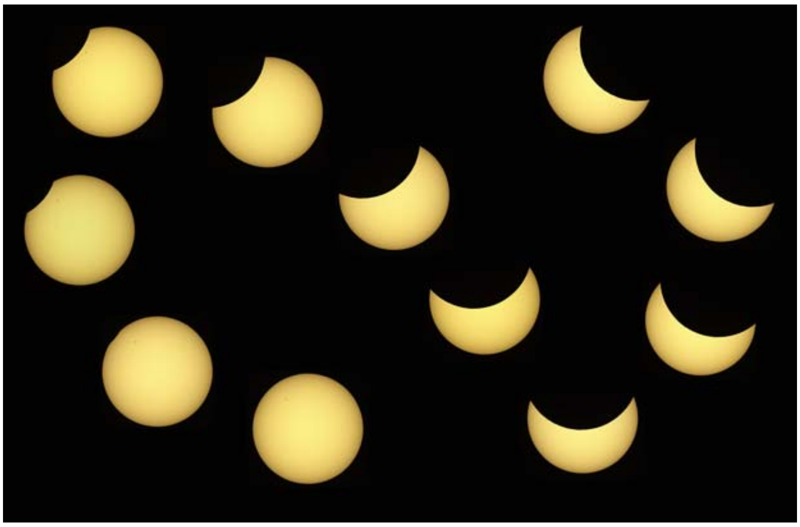
Figures 3 and 4 show the variations of the measured values of *C* and *V* during the partial eclipse of 2015 March 20 observed from our site in Italy. In the case of the instant of the first contact, *Ti*, and the instant of the last contact, *Tf*, the two disks form external tangents, where *C* is zero and *V* is the same as the solar diameter. The maximum phase happens in the instant, *Tm*, when *C* is maximum and *V* minimum.

The problem of determining the instants *Ti*, *Tm* and *Tf* during an eclipse, whether total, partial or annular, can be resolved by analysing the intersection between two circumferences (the Sun and the Moon) which have constant radii (Figure 2). The solar disk is considered as fixed in the reference system of the telescope, whilst the lunar disk appears mobile.

We can obtain a mathematical formula that expresses the length of the chord according to the time if we make the following simple assumptions:

**a) The visible disks are perfectly circular and have constant radii.**
In reality, however, the apparent lunar radius is slowly varying as the eclipse progresses. For example during the solar eclipse of 2005 October 3 at Uccle, near Brussels, the radius of the lunar disk was 909.0 arcseconds at the beginning of the (partial) eclipse, but 913.1 arcseconds at the end.

**b) The Moon moves along a straight line in respect to the Sun placed at distance $Y_L$ from the solar centre (Figure 2).**
Actually, the path is slightly curved due to the diurnal motion of the observer (caused by rotation of the Earth).



**Figures 1a, 1b.** Solar eclipse of 2015 March 20. Images from OACL Mosciano Santangelo, Italy (IAU code C96) taken with a Takahashi Sky 90 telescope (90mm aperture) fitted with an Astrosolar filter and a Canon EOS20 Da DSLR camera operating at 100 ISO with 1/1000 sec exposures. *Images by Giorgio Clemente (above) and Sandro Bocci (left).*

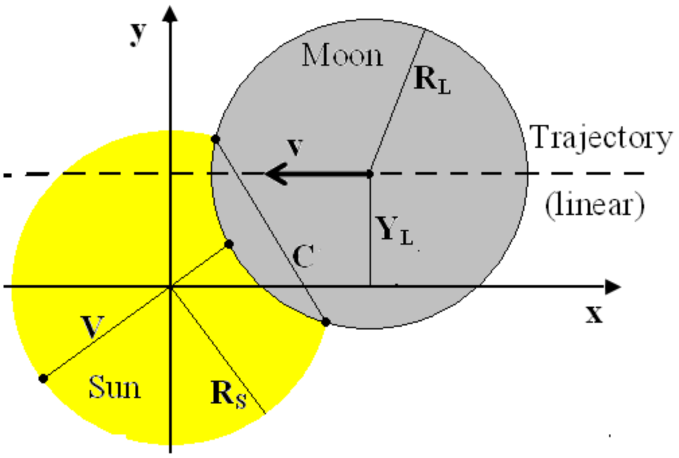



**Figure 2.** In the reference system of the telescope, the solar disk appears stationary whilst the lunar disk appears in rectilinear movement across the solar disk

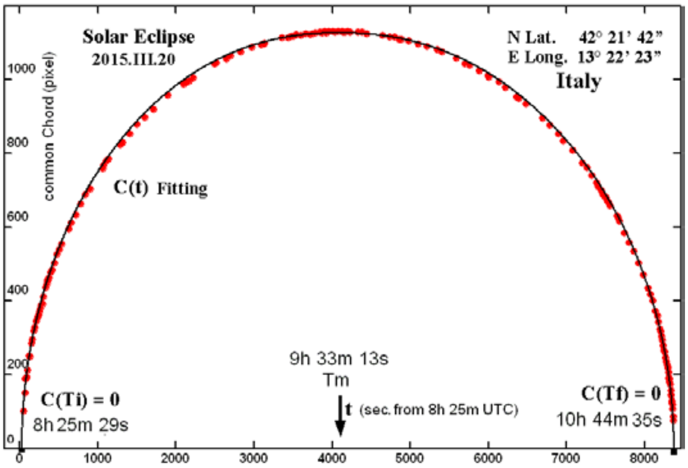



**Figure 3.** Distribution of experimental points (184) and graph (solid line) of the function *C(t)* during the partial eclipse of 2015 March 20 *(Colle Leone Observatory, Italy).*

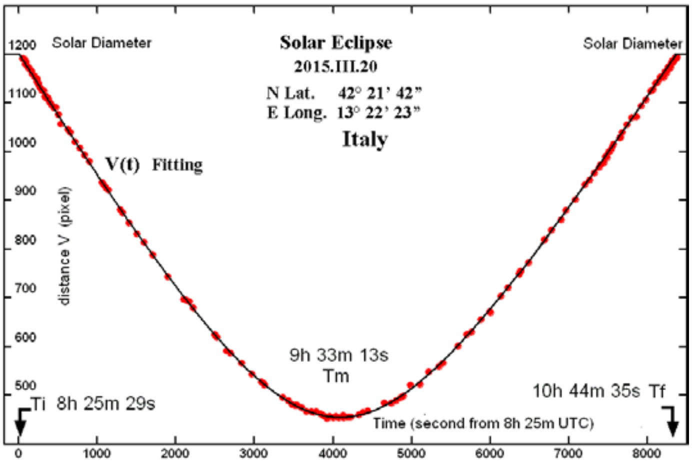



**Figure 4.** Distribution of the measured values of the distance, *V,* between the vertices of the visible arcs and the function *V(t)* during the partial eclipse of 2015 March 20 *(Colle Leone Observatory, Italy).*

**c) The motion of the Moon is uniformly accelerated, so the velocity is v= $v_0$+at (a is the acceleration, t the time, $v_0$ is the velocity in t=0).**

It is true that due to the diurnal motion of the observer, if the eclipse occurs before local noon, the speed of the Moon (in respect to the Sun) decreases, while during an afternoon eclipse the speed increases. However, if the eclipse occurs near noon, the angular speed of the Moon reaches a minimum during the eclipse. Thus the speed cannot be represented by a linear function such as $v=v_0+at$; instead, the formula should contain a term of the second order in $t$.

However, the above caveats on the assumptions (a) to (c) have no real significance for educational purposes, since the effects are very small. Moreover, atmospheric turbulence makes their effects almost negligible.

Therefore we shall use a system composed of only three equations: the equations of the two circumferences and that of the velocity. After many tedious algebraic steps we calculated the coordinates of the intersection points of the lunar and solar circumferences and thus easily obtained (using Pythagoras' theorem) the length of the chord, or rather the theoretical function of the chord $C(t)$ in relation to time:

$$C(t) = \sqrt{-(v_0 t + 0.5at^2)^2 - Y_L^2 + 2(R_L^2 + R_S^2) - \frac{(R_S^2 - R_L^2)^2}{(v_0 t + 0.5at^2)^2 + Y_L^2}} \quad \text{(Eqn 1)}$$

where $t$ is the time counted from the unknown instant, $Tm$ of the maximum phase, and $R_L$ and $R_S$ are the lunar and solar radii respectively.

## Fitting C(t) to the experimental data

Now we have to adapt the theoretical function $C(t)$ in Equation 1 to the measured values of the chord derived from the DSLR images. Rather than applying the usual, and in this case difficult, method of least squares analysis, we can use an empirical technique to fit the function graphically to the data points using commonly available mathematical software (we used *MATLAB*[7]). As such we proceed in the following manner:

1. On some frames we measure the approximate values of the parameters $v_0$, $a$, $Y_L$, $R_L$, $R_S$ and $Tm$ which appear in the theoretical function (Figure 5);
2. Using these we calculate the values of the chord $C(t)$ for example at intervals of a second during the entire duration of the phenomenon;
3. Using the mathematical software we draw a graph of the $C(t)$ values that have just been calculated and we superimpose it on the experimental points;
4. We repeat procedures 2–3 many times, using just one parameter each time, until we obtain a good fit between the values $C(t)$ just calculated and the values of the chord measured in the frames (Figure 3).

**Table 1. Comparison of our measured contact times with predicted times for the partial solar eclipse of 2015 March 20**

| | | *Ti* | *Tm* | *Tf* |
|---|---|---|---|---|
| Ephemeris | [NASA] | 08h 25m 29s | 09h 33m 13s | 10h 44m 36s |
| Observed | [C(t)=0] | 08h 25m 29s | 09h 33m 13s | 10h 44m 35s |

*(All times in UT)*

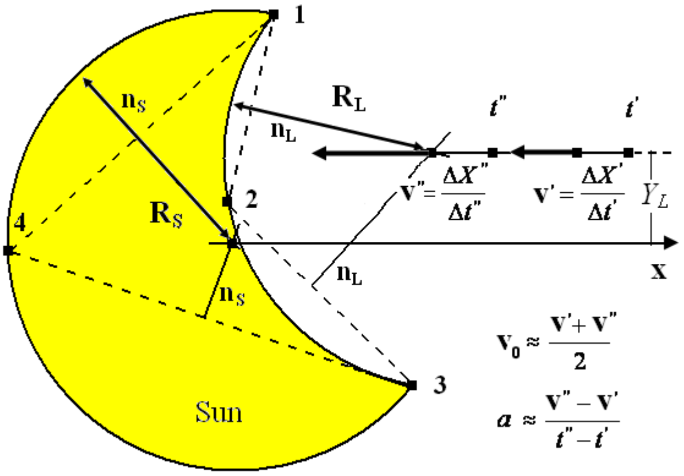



**Figure 5.** Determination of initial values of the parameters $R_L$, $R_S$, $v_0$, $a$, $Y_L$ which appear in the theoretical function *C(t)*. The points 1, 2, 3 (and 1, 4, 3) can be any points on the lunar (and solar) circumference. The lunar (and solar) centre will be the intersection of the perpendicular $n_L$ (and $n_S$) bisectors of the chords between the given points. Δx is the distance travelled by the lunar centre in the time interval, Δt. A correct orientation of images selected is essential for this procedure.

## Experimental results

The result of fitting the experimental data points to the *C(t)* function for the solar eclipse of 2015 March 20 is shown in Figure 3. The curve approximates to a parabola. Thus the values *Ti* and *Tf* are the solutions to the equation *C(t)=0*, while *Tm* derives from the operation of fitting. As shown in Table 1, the timings we obtained are consistent, to within a second, with times predicted by the NASA Eclipse website.[6]

By contrast to a *partial* eclipse, which we have considered until now, in the case of a *total* solar eclipse the graph of the chord is very different to a parabola (Figure 6 and 7). Here, the common chord reaches two maximum values (one *C'* just before totality and one *C''* just after, in the instants *T'* and *T''* respectively) and goes to zero at the instants of the four contacts. Thus, the function *C(t)* adapts itself very well to the experimental data from partial and total eclipses (and it can also be applied to annular eclipses). Figure 7 presents our measurements made during the total eclipse of 2006 March 29, observed from the Libyan desert (unfortunately the precise position of the observing site was not recorded so we cannot compare our measured contact times to predicted times).

Similar developments and considerations apply to the function *V(t)*, which provides the dependency of the distance between the vertices during the eclipse. Given space limitations, we shall not develop the analysis of this function here. Rather, we assign this to the interested reader. Figure 4 shows the resulting graph of *V(t)* and the experimental points.

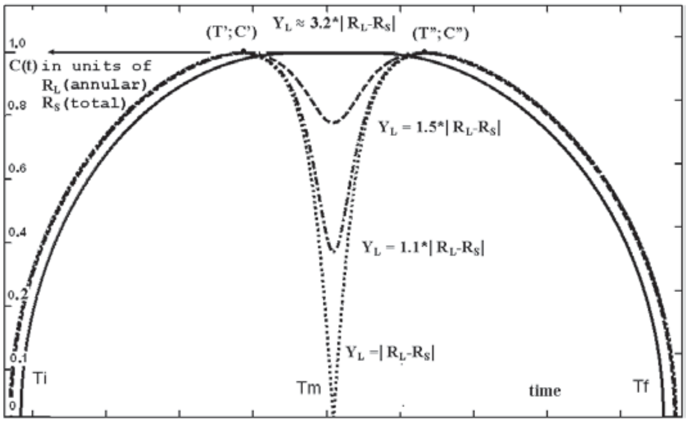



**Figure 6.** Plot of four functions *C(t)* in one graph, with different value ranges $Y_L$. The value of the parameter $Y_L$ is a function of the position with respect to the centre line. For a partial eclipse, $Y_L > |R_L - R_S|$ on the outside of the totality zone. For a total eclipse inside the totality zone, $Y_L \leq |R_L - R_S|$, see Figure 7.

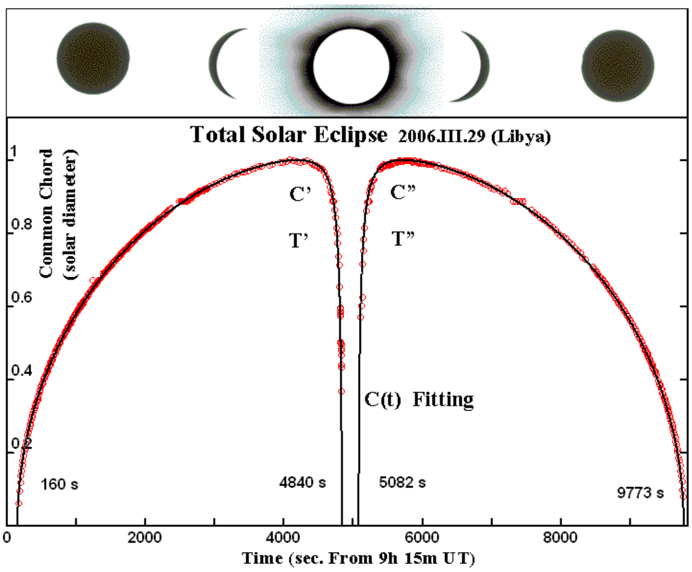



**Figure 7.** Fitting of *C(t)* in the case of a total eclipse ($R_L \geq R_S$), observed within the path of totality $Y_L \leq |R_L - R_S|$. The measured data are for the total eclipse of 2006 March 29, observed from the Libyan desert.

**Table 2. Measured and predicted times for the transits of Venus of 2004 and 2012**

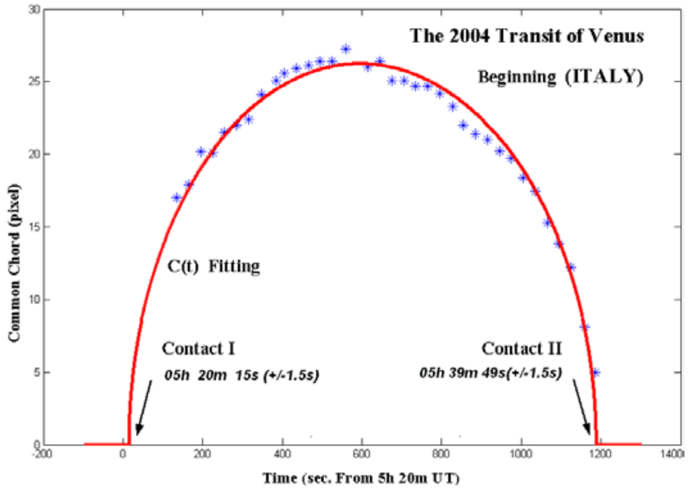
*The predicted times are for the cities of Avezzano and L'Aquila where the telescopes were placed to observe the events for meteorological reasons. Ephemeris= NASA prediction; measured, C(t)=0.*

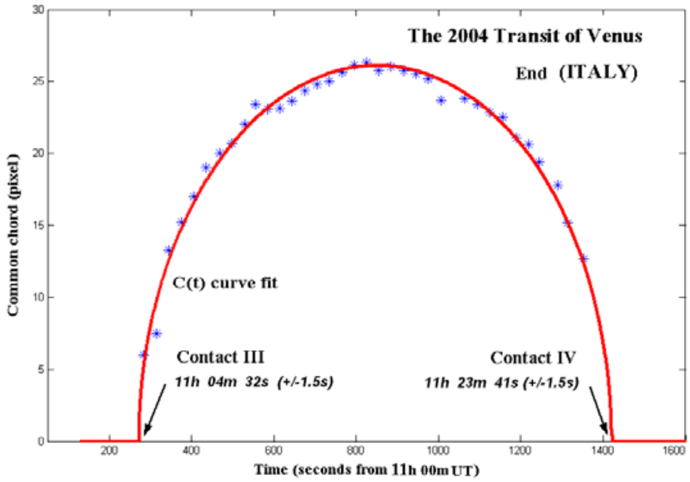
| | *Contact I* | *Contact II* | *Contact III* | *Contact IV* |
|---|---|---|---|---|
| *Venus Transit 2004 June 8 (Avezzano)* | | | | |
| Ephemeris | 5h 20m 11s | 5h 39m 55s | 11h 04m 33s | 11h 23m 53s |
| C(t)=0 | 5h 20m 15s | 5h 39m 49s | 11h 04m 32s | 11h 23m 41s |
| *Venus Transit 2012 June 6 (L'Aquila)* | | | | |
| Ephemeris | – | – | 4h 37m 47s | 4h 55m 26s |
| C(t)=0 | – | – | 4h 37m 48s | 4h 55m 32s |

## Application of the method to planetary transits

A similar analysis of the function *C(t)* is also valid in the case of planetary transits across the Sun. Thus, Figures 8a, 8b and 9 show the *C(t)* graph adapted to the values of the chords measured during the transits of Venus in 2004 and 2012. Our measured contact times (Table 2) show encouraging consistency with the times predicted by the NASA ephemeris.

Unfortunately it is of course a very long time until the next Venus transit. We consider that the same approach would be very difficult for a transit of Mercury, because of the very small size of the planetary disk, and the black drop effect as it enters onto the solar surface is very pronounced, which




**Figures 8a, 8b.** Fitting of *C(t)* in the case of the Venus transit of 2004 June 8. The scatter in the experimental points is due to bad seeing as a result of atmospheric turbulence

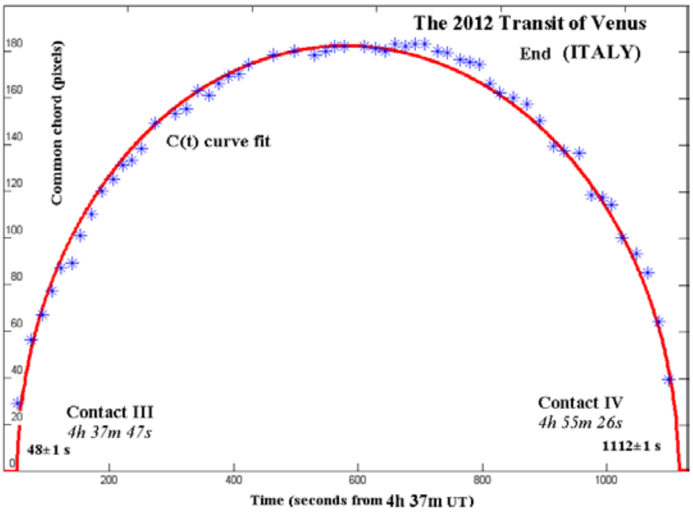



**Figure 9.** Fitting between the theoretical function *C(t)* and 56 experimental points for the Venus transit of 2012 June 6.

will make measurements extremely challenging if not impossible. Nevertheless, the interested reader may like to attempt it during the next transits of 2016 May 9 or 2019 November 11.

## Conclusion

Nowadays, the recording of eclipses via the ubiquitous DSLR camera has become routine. We have presented a method by which contact times can be obtained by careful analysis of such images (video camera frames can of course be analysed in the same way). We suggest that eclipse imagers might like to perform such analyses on their images to obtain accurate values of the contact times as seen from their locations. This would add even more interest to the observation of some of nature's most compelling spectacles.

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**Address:** Astron. Observ. 'Colle Leone' (OACL.net), 64023 Mosciano Sant'angelo, Italy *[cross19@libero.it]*.

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7 **www.mathworks.com/products/matlab/**

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